UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2020

MATHEMATICS

PAPER-VIII (New Syllabus)

Time Allotted: 1 Hour Full Marks: 25

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

GROUP-A

1. Answer the following questions: 2+1+2=5

(a) Find the equation of the circle cutting orthogonally the set of three circles $x^2+y^2-2x+3y-7=0$, $x^2+y^2+5x-5y+9=0$ and $x^2+y^2+7x-9y+29=0$.

(b) Find the point where the straight line joining the points (2, –3, 1) and (3, –4, –5) cuts the plane $3x+y+z=8$.

(c) A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=2$.

2. Answer any ***one*** of the following questions: 3×1 = 3

(a) Prove that the locus of a line which meets the lines $y=mx$, $z=c$ and $y=-mx$, $z=-c$ and which intersects the hyperbola $xy=c^2$, $z=0$ is $(cmx-yz)\,(mxz-cy)+m(c^2-z^2)^2=0$. 3

(b) If the edges of a rectangular parallelepiped be a, b, c then show that the angles between the four diagonals are given by $\cos^{-1}\left(\frac{a^2\pm b^2\pm c^2}{a^2+b^2+c^2}\right)$. 3

(c) If the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$, then find the equation of other two. 3

3. Answer any ***one*** of the following questions: 5×1 = 5

(a) Show that the feet of the normals from the point (α, β, γ) to the paraboloid $x^2+y^2=2az$ lie on the sphere $x^2+y^2+z^2-z(\alpha+\gamma)-\frac{y}{2\beta}(\alpha^2+\beta^2)=0$, where $\beta\neq 0$. 5

(b) Show that the equation to the plane containing the straight line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if $2d$ be the shortest distance between the lines, then show that 5

$$\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

(c) Find the equation of the cylinder whose generators touch the sphere $x^2+y^2+z^2=4$ and are perpendicular to the plane $x+y-2z=8$. 5

GROUP-B

4. Answer the following questions: 2+2+1=5

(a) Find the differential equation of all spheres of radius *a*, having centre in the *xy*-plane.

(b) Eliminate the functions *f* and *F* from $y=f(x-at)+F(x+at)$.

(c) Find $L^{-1}\left\{\frac{4}{p(p-2)}\right\}$.

5. Answer any ***one*** of the following questions: 2×1 = 2

(a) Solve the differential equation $\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}-4xy=0$ in series near $x=0$. 2

(b) Find the eigenvalues and eigenfunction for the differential equation 2

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0,\ (\lambda>0),$$

satisfying the boundary conditions $y(1)=0$ and $y(e^{\pi})=0$.

(c) Solve, by using Laplace transform, the equations 2

$$(D-2)x+3y=0,$$
$$2x+(D-1)y=0,\ t>0$$

and $D\equiv\frac{d}{dt}$ given that $x(0)=8$ and $y(0)=3$.

6. Answer any ***one*** of the following questions: 5×1 = 5

(a) Find the equation of an integral surface given by the differential equation. 5

$$2y(z-3)p+(2x-z)q=y(2x-3),$$

which passes through the circle $z=0$, $x^2+y^2=2x$.

(b) Solve $(D^2-2D)y=e^x\cos x$ by the method of variation of parameter. 5

(c) Solve using Laplace transform: 5

$$\{tD^2+(1-2t)D-2\}y=0,\ D\equiv\frac{d}{dt} \text{ given } y(0)=1 \text{ and } y'(0)=2.$$

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