'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2021

CC5-PHYSICS

MATHEMATICAL PHYSICS-II

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer any ***five*** questions: $1\times5 = 5$

(a) What do you mean by singular points of second order differential equations? 1

(b) What are the Dirichlet's conditions? 1

(c) What is the Fourier transform of delta function? 1

(d) Under which condition Poisson's equation reduces to Laplaces equation? 1

(e) Define gamma functions and write its integral formula. 1

(f) Solve $\frac{\partial^2 z}{\partial x \partial y} = 0$. 1

(g) If $\int_{-1}^{+1} P_n(x)dx = 2$, then $n = ?$ 1

(h) Identify the odd function 1

(i) x^2 (ii) $\sec x$ (iii) $\cos x$ (iv) $\tan x$

GROUP-B

Answer any *three* questions $5\times3 = 15$

2. If a string is plucked at its mid-point by the displacement 'h', find the expression for displacement of any point on the string. 5

3. Show that $2^n \Gamma(n+\frac{1}{2}) = 1\cdot3\cdot5\cdot \cdots (2n-1)\sqrt{\pi}$. 5

4. Find the Fourier transform of the Gaussian Probability function: 5

$$f(x) = Ne^{-\alpha x^2} \quad (N, \alpha \text{ are constants})$$

5. Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}dx}{(a+x)^{m+n}} = \frac{\Gamma m\ \Gamma n}{a^n(1+a)^m \Gamma(m, n)}$ 5

6. Evaluate $\int_0^\infty x^4 e^{-x} dx$ and $\int_0^\infty e^{-4x} x^{5/2} dx$ $2\frac{1}{2}+2\frac{1}{2}$

GROUP-C

Answer any *two* questions 10×2 = 20

7. (a) Find the inverse sine transformation of $e^{-\lambda n}$. 3

(b) Prove that $\int_{-1}^{+1} P_n(x)P_m(x)dx = \frac{2}{2m+1}\delta_{m,n}$ 4

where $\delta_{m,n}$ is Kronecker delta symbol.

(c) Show that $P_{2n}(0) = (-1)^n \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot 2n}$. 3

8. (a) Explain physical significance of Fourier transform of a function with example. Check the possibilities of Fourier series of $f(x) = x(-\infty \le x \le \infty)$ on the basis of Dirichlet conditions. 2+3

(b) Find the Fourier series for the following function 5

$$f(x) = x + x^2,\ -\pi < x < +\pi$$

9. (a) For the Legendre's polynomials, prove the following recurrence relation 4

$$(l+1)P_{l+1}(x) - (2l+1)x\,P_l(x) + l\,P_{l-1}(x) = 0$$

(b) Show that $e^{x(t-1/t)/2} = \sum_{n=-\infty}^{+\infty} J_n(x)t^n$ and hence deduce the relation $J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)d\theta$ where n is any integer. 3+3

10.(a) Solve Laplace's equation in spherical polar co-ordinates. Hence write down the values of first four spherical harmonics. Discuss the properties of spherical harmonics. 4+1+1

(b) Using variational calculations show that the shortest distance joining two points is a straight line. 4

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