'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2021

DSE-P1-MATHEMATICS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

The question paper contains DSE1A and DSE1B. Candidates are required to answer any *one* from the *two* DSE1 courses and they should mention it clearly on the Answer Book.

DSE1A

PROBABILITY AND STATISTICS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4 = 12$

(a) If X be a continuous random variable with uniform distribution having mean 1 and $\operatorname{var}(X)=1$, find $P(X\leq 0)$.

(b) If the two regression lines are $x+6y=6$ and $3x+5y=10$, find the correlation coefficient.

(c) A random variable X has the density function $f(x)=\dfrac{a}{x^2+1}$, $-\infty<x<\infty$. Find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(d) Show that the sample mean based on a simple random sample with replacement is an unbiased estimator of the population mean.

(e) A coin is tossed 10 times. Find the probability of getting exactly 6 heads.

(f) A dice was thrown 400 times and 'six' resulted 80 times. Do the data justify the hypothesis of an unbiased dice?

GROUP-B

2. Answer any ***four*** questions from the following: $6\times 4 = 24$

(a) In a bolt factory, the machines M_1, M_2, M_3 manufacture respectively 25, 35 and 40 percent of the total product. Of their output 5, 4 and 2 percent respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine M_3?

(b) If X and Y be independent binomial variables with parameters (m, p) and (n, p) respectively, show that their sum $X+Y$ has a binomial distribution with parameters $(m+n, p)$.

(c) A rod of length 'a' is broken into three parts at random. Find the probability of their forming a triangle.

(d) Let X and Y have joint density function 4+2

$$f(x, y)=\begin{cases} \dfrac{e^{-(x+y)}x^3y^4}{\Gamma(4)\,\Gamma(5)} & ; \quad x>0\,,\ y>0 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Find the density function of $U=\dfrac{X}{X+Y}$ and $\operatorname{var}(U)$.

(e) A random sample of size 20 from a normal population gives a sample mean of 42 and sample standard deviation of 6. Test the hypothesis that the population mean is 44. State clearly the alternative hypothesis you allow for and the level of significance adopted.

(f) State and prove Tchebycheff's inequality. 2+4

GROUP-C

3. Answer any ***two*** questions from the following: 12×2=24

(a) (i) If X be a binomial (n, p) variate, then show that 6+6

$$P(X \le k)=\frac{\int_0^q x^{n-k-1}(1-x)^k\,dx}{\int_0^1 x^{n-k-1}(1-x)^k\,dx}$$

where $q=1-p$ and k is an integer such that $0 \le k \le n-1$.

(ii) If X be a continuous random variable having spectrum $(-\infty, \infty)$ and distribution function $F(x)$, show that

$$E(X)=\int_0^\infty \{1-F(x)-F(-x)\}\,dx$$

provided $x\{1-F(x)-F(-x)\} \to 0$ as $x \to \infty$.

(b) A random sample of 100 articles taken from a large batch of articles contains 5 defective articles. 6+6

(i) Set up 96 percent confidence limits for the population proportion of defective articles in the batch.

(ii) If the batch contains 2696 articles set up 95% confidence interval for the proportion of defective articles.

(c) (i) Let $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of random variables such that $S_n=X_1+X_2+\ldots\ldots+X_n$ has a finite mean M_n and a finite variance B_n for all n then prove that 6+6

$$\frac{S_n-M_n}{n} \xrightarrow{\text{in P}} 0 \text{ as } n\to\infty \ ; \text{ if } \frac{B_n}{n^2}\to 0 \text{ as } n\to\infty$$

(ii) Show that if X is a random variable having the Poisson distribution with parameter μ and $\mu \to \infty$ then the moment generating function (m.g.f.) of $Z = \dfrac{X-\mu}{\sqrt{\mu}}$ approximate to m.g.f. of the standard normal distribution.

(d) (i) Let X be distributed in the Poisson form. If $P(X=1)=P(X=2)$ find the value of $P(X=0 \text{ or } 1)$. Also, find $E(X)$. 6+6

(ii) The marginal distribution of X and Y are given in the following table:

y \ x	5	7	Total
3	?	?	1/3
6	?	?	2/3
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

If $\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$ is known to be $\left(-\frac{1}{2}\right)$, obtain the cell probabilities.

DSE1B

LPP

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find the Basic Feasible solution of the following set of equation: 3

$$4x_1 + 2x_2 - 3x_3 = 1$$
$$6x_1 + 4x_2 - 5x_3 = 1$$

(b) Solve graphically the following LPP: 3

Maximize $Z = x_1 + 3x_2$

Subject to $3x_1 + 6x_2 \le 8$

$5x_1 + 2x_2 \le 10$

$x_1,\ x_2 \ge 0$

(c) Find the dual of the following LPP: 3

Maximize $Z = 3x_1 + 2x_2$

Subject to $3x_1 + 4x_2 \le 22$

$3x_1 + 2x_2 \le 16$

$x_1,\ x_2 \ge 0$

(d) Prove that in E^2, the set $X = \{(x_1, x_2) / x_2^2 \ge 4x_1\}$ is not a convex set. 3

(e) Show that the vectors $\bar{a}_1 = (1, 0, 0)$, $\bar{a}_2 = (0, 1, 0)$ and $\bar{a}_3 = (1, 5, 3)$ form a basis for E^3. Indicate whether the vector $\bar{a}_3$ can be replaced by new vector $(0, 1, 1)$ to form a basis of E^3. 3

(f) Solve the 2×2 game: 3

Player B

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} -1 & 6 \\ 5 & 2 \end{bmatrix} \end{array}$$

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Solve the LPP: 6

Maximize $Z = 2x_1 + 5x_2$

Subject to $2x_1 + x_2 \geq 12$

$x_1 + x_2 \leq 4$

$x_1 \geq 0$ and x_2 is unrestricted in sign.

(b) Find the optimal solution of the following transportation problem. 6

	D_1	D_2	D_3	a_i
O_1	4	3	2	10
O_2	1	5	0	13
O_3	3	8	6	12
b_j	8	5	4	

(c) Use duality to find the optimal solution (if any) of the following LPP: 6

Minimize $Z = 15x_1 + 10x_2$

Subject to $3x_1 + 5x_2 \geq 5$

$5x_1 + 2x_2 \geq 3$

$x_1,\ x_2 \geq 0$

(d) Solve the following LPP by Big M-method: 6

Maximize $Z = 2x_1 - 3x_2$

Subject to $-x_1 + x_2 \geq -2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

$x_1,\ x_2 \geq 0$

(e) (i) Write the mathematical form of an assignment problem. 3+3=6

(ii) Show that the solution of a Transportation problem is never unbounded.

(f) Solve the following 4×4 game and prove that there are two saddle points:

	B_1	B_2	B_3	B_4
A_1	4	2	3	5
A_2	– 2	– 1	4	– 3
A_3	5	2	3	3
A_4	4	0	0	1

GROUP-C

3. Answer any ***two*** questions from the following: 12×2=24

(a) (i) Solve the following LPP by Simplex method: 6+6

Maximize $Z = -x_1 + 3x_2 - 2x_3$

Subject to $3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1,\ x_2,\ x_3 \ge 0$

(ii) Solve the following LPP by two phase method:

Minimize $Z = 4x_1 + x_2$

Subject to $x_1 + 2x_2 \le 3$

$4x_1 + 3x_2 \ge 6$

$3x_1 + x_2 = 3$

$x_1,\ x_2 \ge 0$

(b) (i) Find the optimal solution of the dual of the problem. 6+6

Maximize $Z = 5x_1 + 12x_2 + 4x_3$

Subject to $x_1 + 2x_2 + x_3 \le 5$

$2x_1 - x_2 + 3x_3 = 2$

$x_1,\ x_2,\ x_3 \ge 0$

from the Simplex table of the primal.

(ii) Consider the LPP:

Maximize $Z = x_1 + 5x_2 + 3x_3$

Subject to $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1,\ x_2,\ x_3 \ge 0$

Let x_4 be an artificial variable in the second constraint equation. Solve the problem by using x_3 and x_4 for starting basic solution.

(c) (i) Solve the assignment problem where matrix is a profit matrix. 6+6

	1	2	3	4	5
A	10	2	– 7	8	6
B	0	8	– 5	5	4
C	8	3	2	1	5
D	1	1	4	2	3
E	8	2	1	– 1	4

(ii) The pay-off matrix of a rectangular game is

	B_1	B_2	B_3	B_4	B_5
A_1	10	5	5	20	4
A_2	11	15	10	17	25
A_3	7	12	8	9	8
A_4	5	13	9	10	5

State whether the players will use pure and mixed strategies. What is the value of the game?

(d) (i) Prove that the set of all feasible solutions of an LPP is a convex set. 4+8

(ii) For the following pay-off table, transform the zero sum game into an equivalent LPP and solve it by Simplex method:

		B		
		B_1	B_2	B_3
A	A_1	9	1	4
	A_2	0	6	3
	A_3	5	2	8

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