'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2021

CC11-PHYSICS

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer any ***five*** of the following: 1×5 = 5

(a) Find out the value of A if the wave function $\psi(x) = Ae^{ikx}$ is normalized over the region $-a \le x \le a$.

(b) Employing the uncertainty principle estimate the maximum kinetic energy of an election confined within a region whose size is $l = 0.2\,\text{nm}$.

(c) Let f_1 and f_2 be two orthonormalized states of two electrons. Construct the two-electron state keeping Pauli's exclusion principle in mind.

(d) Tick out the correct answer(s) and explain it.

Potential of a free particle is

(i) maximum
(ii) minimum
(iii) zero
(iv) infinity
(v) uniform

(e) Operators $\hat{A}$ and $\hat{B}$ are the quantum operators representing two observables A and B of a physical system. Then what must be true of the eigen values of the commutator $[\hat{A}, \hat{B}]$ so that the system can have definite values of both A and B simultaneously.

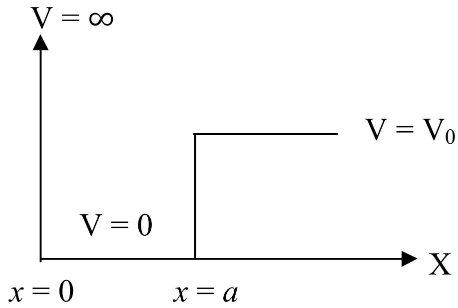
(f) If $\dfrac{1}{\sqrt{V}}e^{i\vec{k}\cdot\vec{r}}$ represents the state of a particle in the co-ordinate space, what will be the state in the momentum space?

(g) What is the origin of fine structures in the hydrogen atom spectrum?

GROUP-B

Answer any *three* of the following 5×3 = 15

2. A particle is located in a one dimensional well with an infinite potential wall on one side and a finite potential wall on the other side as shown in the figure below 3+2



(a) For $E < V_0$, write down and solve the Schrodinger equation for (i) the region inside the well and (ii) the region outside the well.

(b) Apply the boundary conditions at $x = 0$ and $x = a$ to obtain an equation that defines the allowed values of the energy E.

3. (a) Develop the Schrodinger equation of a system consisting of two identical bosons confined in a one dimensional box. 3+2

(b) What is the ground state energy of a one dimensional box containing three electrons?

4. For a large, value of the external magnetic field B (applied in z directions) the Zeeman Splitting is given by 3+2

$$\Delta E = -\langle \mu_2 \rangle B$$

when $\langle \mu_2 \rangle$ is the expectation value of electron's dipole moment in the z-direction.

(a) Show that

$$\langle \mu_2 \rangle = \frac{e\hbar}{\partial m_e}[m_l + \partial m_s]$$

where does the factor of ∂ come from?

(b) Into how many states does the 3D state of hydrogen split in a strong external magnetic field?

5. Determine the expectation value of the potential energy of a linear harmonic oscillator with respect to its n^{th} eigen-state. Comment on the result that you get. 5

6. Consider a particle of mass M moving on a ring of radius R with the centre at the origin. The ring lies in the x-y plane. Find out the allowed energy values of the particle. 5

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. A particle of mass m is confined within an infinite potential well. Find out the energy eigenvalue and normalize the wave function. Derive the expectation value of position in the ground state of the system. Show that the eigenfunctions corresponding to different states are mutually orthogonal. 3+2+2+3

8. (a) What do you understand by L-S and J-J coupling? 2+2+3+3

   (b) What are the values of total angular momentum J and orbital angular momentum L for the atomic state ${}^2D_{5/2}$?

   (c) Suppose the Hamiltonian for a single particle is given by,

   $$H = A + B\vec{L}\cdot\vec{S} + C\,\vec{L}\cdot\vec{L}$$

   where A, B and C are constants. A spin-$\frac{1}{2}$ particle is in the D-state. What will be the possible values of J? Show that the eigenstates of $\hat{J}$ are also eigenstates of the above Hamiltonian.

   (d) Find out the Lande's g-factor for an s-electron.

9. (a) Distinguish between the normal and anomalous Zeeman effect. 1+4+3+2

   (b) Obtain the energy splitting formula for the anomalous Zeeman effect and show that the same can be applied to the normal Zeeman effect.

   (c) Explain how many spectral lines are observed for the $(1s^2 3p)\ {}^2P_{1/2} \rightarrow (1s^2 2s)\ {}^2s_{1/2}$ transition in Lithium. Is this a normal or an anomalous Zeeman effect? Justify your answer.

   Draw an energy level diagram showing the splitting of levels and the allowed transitions.

   (d) What is the significance of π and σ lines?

10. (a) Solve the eigenvalue problem for a simple harmonic oscillator. 5+2+1+2

    (b) Draw a sketch of $\psi_4(x)$ against x superposed on the $V(x)$ vs. x plot for the above problem.

    (c) What is the significance of zero point energy?

    (d) Find out the zero point energy of a quantum simple pendulum of length 1 mt and a bob of mass 0.1 kg.

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