'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2022

PHYSICS

PAPER-IX

Time Allotted: 4 Hours | Full Marks: 90

The figures in the margin indicate full marks.
All symbols are of usual significance.

Answer question no. 1 and *five* other from the rest, taking at least *one* from each group but not more than *two* from a group

1. (a) Show that the momentum operator $-i\hbar\vec{\nabla}$ is a Hermitian operator. 3

   (b) Obtain the expression for the probability current density for the wave function $\psi(r) \propto e^{ikr}/r$. 3

   (c) Prove that the eigenfunctions belonging to distinct eigenvalues of a Hermitian operator are mutually orthogonal. 3

   (d) Find out the eigenfunction and eigenvalue of the operator $-i\dfrac{d}{dx}$, if the eigenfunction $\psi(X)$ follows the condition $\psi(X) = \psi(x+a)$. 3

   (e) The series limit of the Balmer series is at 3646Å. Calculate the wavelength of the first line of the series. 3

GROUP-A

(ATOMIC PHYSICS)

2. (a) Describe Thomson's method for the determinates of e/m of an electron. What are the drawbacks of this method? 5+1

   (b) What is the Bohr magneton? What modification is introduced in Bohr's theory of an atom due to the finite mass of a nucleus? 2+3

   (c) Find out the expressions for the magnetic dipole moment due to the orbital motion of an electron and the corresponding gyromagnetic ratio. 4

3. (a) What are normal and anomalous zeeman effects? Outline the theory of normal Zeeman effect. 3+3

   (b) State and explain Pauli's exclusion principle. How do you explain the D_1 and D_2 doublet of sodium on the basis of electron spin? 3+3

   (c) The ionisation potential of an atom is 13.6 V. With what minimum velocity should an electron strike so that it may be ionised? 3

4. (a) Describe the Stern and Gerlach's experiment. Explain the experimental results. 3+1

(b) State and explain Moseley's law and discuss its importance. Show that this law can be deduced from modified Bohr's theory. 4+2

(c) Show that the magnetic moment of an atom can be expressed as 5

$$\mu_j = \mu_B g\sqrt{J(J+1)}\,,$$

where μ_B is the Bohr magneton, g is Lande g factor and J is the total angular momentum quantum number.

GROUP-B
(QUANTUM MECHANICS)

5. (a) Find out the commutation relation $[e^{ikx}, \hat{p}_x]$, where x and p_x have their usual significance. 3

(b) The ground state wave function of hydrogen atom is given by $\psi(r) = \sqrt{\frac{1}{\pi a^3}}e^{-r/a}$, 2+2

where r measures the distance from the nucleus and 'a' is the Bohr radius (constant). Write down the probability that the electron lies between r and $r+dr$. Hence show that the most probable value of r is a.

(c) Prove the following operator relation 2+4

$$\left[\frac{1}{x}\frac{d}{dx}x\right]^2 = \frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx}$$

Verify whether the above operator is hermitian or not.

(d) State and explain Heisenberg's energy-time uncertainty principle. 2

6. (a) Set up the Schrödinger equation for a linear harmonic oscillator. Solve it to obtain the energy eigenvalues and eigenfunctions. Highlight the physical significance of the asymptotic behaviour of the wave functions. 2+8

(b) What is the significance of the zero point energy? Calculate the zero point energy of a mass 1.67×10^{-24} gm connected to a fixed point by a spring with a force constant 10^4 dyne/cm. 2+3

7. (a) A hydrogen atom is in a state represented by the wave function 6

$$\psi(\vec{r}) = \frac{1}{\sqrt{3}}\psi_{200}(\vec{r}) + \sqrt{\frac{3}{2}}\psi_{210}(\vec{r})$$

where $\psi_{nlm}(\vec{r})$ is the energy eigenfunction of the H atom with energy

$$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2 n^2 h^2}\,.$$

Does this state possess definite values of E, L^2 and L_z? Explain. Calculate the expectation values of these quantities with respect to the state $\psi(\vec{r})$.

(b) Using the uncertainty principle estimate (i) the ground state energy of a harmonic oscillator and (ii) the radius of the first Bohr orbit of a hydrogen atom. 2+2

(c) Show that for a particle of rest mass m_0 and wave length λ, the phase velocity of the de Broglie wave is 3+2

$$v_p = c\sqrt{1+(m_0 c\lambda/h)^2}$$

Also show that the corresponding group velocity v_g, is the same as the velocity of the particle.

GROUP-C

(NUCLEAR AND PARTICLE PHYSICS)

8. (a) Explain the terms, mass defect and binding energy of a nucleus. 2+2

(b) Show graphically how the nuclear binding energy per nucleon varies with the mass number. Explain the different parts of the graph. 2+2

(c) A free neutron decays into a proton, an electron and an anti-neutrino. If $M(n)=1.00898\,\text{u}$, $M(p)=1.00759\,\text{u}$ and $M(e)=0.00055\,\text{u}$. Calculate the kinetic energy shared by the electron and the anti-neutrino. 3

(d) What is meant by the activity of a radioactive sample? What are its different units? 2+2

9. (a) Describe the construction and the working principle of a GM counter. How is quenching achieved in it? What are dead time, resolving time and recovery time? 4+2+3

(b) In a radioactive decay process, λ_1 and λ_2 are the decay constants of parent and daughter nuclei respectively. Prove that the time at which the number of daughter nuclei become maximum is given by 4

$$t_m = \frac{\ln(\lambda_2/\lambda_1)}{(\lambda_2-\lambda_1)}$$

(c) Explain the straggling of the range of α particles. 2

10. (a) Write down the Bethe-Weizsacker mass formula and explain the different terms present in it. 5

(b) A beam of monoenergetic γ rays is incident on an Al sheet of thickness 8 cm. The sheet reduces the intensity of the beam to 20% of the original value. Calculate the linear and mass absorption coefficients. [Given the density of $\text{Al} = 2700\ \text{kg m}^{-3}$]. 4

(c) Check whether the following reactions are allowed or forbidden $1\frac{1}{2}\times 4 = 6$

(i) $\mu^- \rightarrow e^- + \nu_e + \bar{\gamma}_e$

(ii) $\bar{\nu}_\mu + p \rightarrow n + \mu^+$

(iii) $\pi^0 \rightarrow \gamma + \gamma$

(iv) $\Lambda^0 + \Xi^- \rightarrow \bar{p} + \Sigma^+ + e^-$

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