'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2022

DSE-P4-MATHEMATICS

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

The question paper contains DSE4A and DSE4B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.

DSE4A

DIFFERENTIAL GEOMETRY

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. For the curve $\vec{r}=(3u,\ 3u^2,\ 2u^3)$, show that radius of curvature $R=\frac{3}{2}(1+2u^2)^2$.

2. Find the equation to the developable surface which has the helix $x=a\cos u$, $y=a\sin u,\ z=cu$ for its edge of regression.

3. Find the length of the curve given as the intersection of the surfaces
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ ,\quad x=a\cosh(z/a)$$
from the point $(a,0,0)$ to (x,y,z).

4. Prove that the geodesic curvature vector of a curve is orthogonal to the given curve.

5. If the n^{th} derivative of $\vec{r}$ with respect to s is given by $\vec{r}^{\,(n)}=a_n\vec{t}+b_n\vec{n}+c_n\vec{b}$, prove that $b_{n+1}=b_n'+ka_n-\tau c_n$.

6. Prove that the curve given by $x=a\sin^2 u,\ y=a\sin u\cos u,\ z=a\cos u$ lies on a sphere.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Show that a curve is a helix if and only if the curvature and torsion of that curve are in constant ratio. 6

8. If the tangent and binormal at any point on a curve make angles θ and ϕ respectively with a fixed direction, then prove that $\frac{\sin\theta}{\sin\phi}\cdot\frac{d\theta}{d\phi}=-\frac{k}{\tau}$. 6

9. (a) Prove that the asymptotic lines are orthogonal iff the surface is minimal. 3

(b) Show that the parametric curve on a surface $r(u, v)=(u\cos v, u\sin v, v)$ are asymptotic line. 3

10. Find the parametric direction and angle between parametric curves. 3+3

11.(a) Find the equation of the tangent plane and normal to the surface $xyz=4$ at the point $(1, 2, 2)$. 3

(b) Prove that the surface $xy=(z-c)^2$ is developable. 3

12.(a) Define first fundamental form. 1

(b) Show that, if θ is the angle at the point (u, v) between the two directions given by $P\,du^2+2Q\,du\,dv+R\,dv^2=0$; then $\tan\theta=\frac{2H(Q^2-PR)^{1/2}}{ER-2FQ+GP}$. 5

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13. Prove that for any curve: 2+2+2+2+2+2

(i) $\vec{r}'\cdot\vec{r}''=0$

(ii) $\vec{r}'\cdot\vec{r}=-\kappa^2$

(iii) $\vec{r}''\cdot\vec{r}'''=\kappa\kappa'$

(iv) $\vec{r}\cdot\vec{r}'^{v}=-3\kappa\kappa'$

(v) $\vec{r}''\cdot\vec{r}'^{v}=\kappa(\kappa''-\kappa^3-\kappa\tau^2)$

(vi) $\vec{r}'''\cdot\vec{r}'^{v}=\kappa'\kappa''+2\kappa^3\kappa'+\kappa^2\tau\tau'+\kappa\kappa'\tau^2$

14.(a) State and prove Serret-Frenet formulae. 6

(b) Find the arc-length parametrization for each of the following curves: 3+3

$$\vec{r}(t)=4\cos t\,\hat{i}+4\sin t\,\hat{j}\ ,\ t\geq 0 \quad \text{and} \quad \vec{r}(t)=(t+3,\ 2t-4,\ 2t),\ t\geq 3$$

15.(a) Show that the parametric curves are orthogonal on the surface 6

$$r=(u\cos v,\ u\sin v,\ a\log\{u+\sqrt{u^2-\alpha^2}\})$$

(b) Find the Principal direction and Principal curvature on a point of the surface 6

$$x=a(u+v),\quad y=b(u-v),\quad z=uv$$

16.(a) Find the involute and evolute of a circular helix. 3+3

(b) Show that the curves $u+v=\text{constant}$ are geodesic on a surface with the metric 6

$$(1+u^2)\,du^2-2uv\,du\,dv+(1+v^2)\,dv^2$$

DSE4B

THEORY OF EQUATIONS

GROUP-A

Answer any *four* questions from the following $3\times 4=12$

1. Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4+x^2+x-1=0$. 3

2. If α, β, γ be the roots of the equation $x^3+3x^2-x+3=0$, find the value of $\sum\frac{1}{\alpha}$. 3

3. Express the polynomial $8x^3+2x+2$ as a polynomial in $2x-1$. 3

4. Find the remainder when $x^{10}+x^7+x^4+x^3+1$ is divided by x^2+1. 3

5. Form a cubic equation with real coefficients whose two of the roots are 1 and $-1-i$. 3

6. If α, β, γ be the roots of the equation $x^3+x-2=0$, then find the equation whose roots are $\alpha+3,\ \beta+3,\ \gamma+3$. 3

GROUP-B

Answer any *four* questions from the following $6\times 4=24$

7. Find the range of values of k for which the equation $x^4-26x^2+48x-k=0$ has four unequal roots. 6

8. Calculate Sturm's function and locate the position of real roots of the equation $x^4-x^2-2x-5=0$. 6

9. If α, β are the roots of the equation $t^2+2t+4=0$ and m is a positive integer, then prove that $\alpha^m+\beta^m=2^{m+1}\cos\dfrac{2m\pi}{3}$. 6

10.(a) Prove that the equation $(x+1)^4=a(x^4+1)$ is a reciprocal equation if $a\neq 1$ and solve it if $a=-2$. 4

(b) If $x^3+3px+q$ has a factor of the form $(x-\alpha)^2$, show that $q^2+4p^3=0$. 2

11. If $\alpha+\beta+\gamma=1$, $\alpha^2+\beta^2+\gamma^2=3$ and $\alpha^3+\beta^3+\gamma^3=7$, find the value of $\alpha^4+\beta^4+\gamma^4$. 6

12.(a) Solve: $x^3-18x-35=0$ 4

(b) If α is an imaginary root of the equation $x^{11}-1=0$, prove that $(\alpha+2)(\alpha^2+2)\,.......\,(\alpha^{10}+2)=\dfrac{2^{11}+1}{3}$. 2

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Solve the equation $x^4+12x^3-18x^2+6x+9=0$, given that the ratio of two roots is equal to the ratio of other two roots. 6

(b) Solve by Ferrari's method: $x^4-4x^3+5x+2=0$ 6

14.(a) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\alpha\beta+\beta\gamma,\ \beta\gamma+\gamma\alpha,\ \gamma\alpha+\alpha\beta$. 6

(b) State Fundamental Theorem of classical algebra. If α is a root of the equation $\dfrac{1}{x-1}+\dfrac{2}{x-2}+\dfrac{3}{x-3}+\dfrac{4}{x-4}=x-5$, prove that α is a non-zero real number. 2+4

15.(a) Find the limits of the negative roots of the equation 6

$$30x^4+41x^3-136x^2+31x+12=0$$

(b) Express the polynomial $x^4+3x^3+5x^2+3x+1$ as a polynomial in $(x-3)$ and $(x+2)$. 6

16.(a) Find the relation among the coefficients of the equation $x^4+px^3+qx^2+rx+s=0$ if its roots α, β, γ and δ be connected by the relation $\alpha+\beta=\gamma+\delta$. 6

(b) Solve: $3x^6+x^5-27x^4+27x^2-x-3=0$ 6

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