'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2022

## CC14-PHYSICS

### STATISTICAL MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
*All symbols are of usual significance.*

### GROUP-A

1. Answer any ***five*** questions of the following: 1×5 = 5

(a) Write down the Additive law of probability.

(b) Which statistics is obeyed by an atomic nucleus?

(c) What is meant by 'Thermodynamic limit'?

(d) Sketch a plot of $C_V$ vs. $T$ for an ideal Bose gas highlighting the important features.

(e) An energy level is 3 fold degenerate. In how many ways can two Maxwell-Boltzmann particles be distributed over them?

(f) Identical particles can be considered as distinguishable if

(i) $n\lambda^3 >> 1$, (ii) $n\lambda^3 \approx 1$, (iii) $n\lambda^3 << 1$, (iv) None of these

(g) Chemical potential of a bosonic system cannot be negative. — Explain.

(h) (3, 1, 1) and (2, 3, 0) are the two macrostates of a system of five particles corresponding to energy $9E$. Which of the above two corresponds to most probable distribution?

### GROUP-B

**Answer any *three* questions of the following** 5×3 = 15

2. (a) Derive Wien's displacement law of black body radiation from Planck's law. 3

(b) A body at 1500 K emits maximum energy of radiation at a wavelength of 2000 nm. If the sun emits maximum energy of radiation at 550 nm, what is the temperature of the sun? 2

3. (a) What do you mean by electron gas in a metal? Why is it called a 'highly degenerate Fermi system'? $1\frac{1}{2}+1\frac{1}{2}$

(b) The number of conduction electrons per c.c. in Beryllium is $24.2 \times 10^{22}$ and in Cesium it is $0.91 \times 10^{22}$. If the Fermi energy of conduction electrons in Beryllium is 14.44 eV, calculate the Fermi energy in Cesium. 2

4. (a) Explain the concept of phase space and phase trajectory. 2

(b) Find the phase space trajectory of a one dimensional oscillator having energy $E$, mass $m$ and frequency $\nu$. 2

(c) Calculate the number of phase cells available. 1

5. Show that, under MB statistics the number of molecules of an ideal gas in equilibrium at temperature $T$ having momentum in the range from $p$ to $p+dp$ is given by, $h(p)\,dp = \dfrac{4\pi N}{(2\pi mkT)^{3/2}} \cdot p^2 e^{-\frac{p^2}{2mkT}}\, dp$. $3\frac{1}{2}+1\frac{1}{2}$

Hence find the expression of most probable momentum.

6. Show that, for a two dimensional free electron gas, the number of electrons per unit area is given by, $n = \dfrac{4\pi mkT}{h^2} \ln(E^{E_\mathrm{F}/kT} + 1)$. 5

## GROUP-C

**Answer any *two* questions of the following** 10×2 = 20

7. (a) Establish Gibbs paradox for mixing of two ideal gases, assuming appropriate expression for entropy. 3

(b) Discuss Gibbs' solution of the paradox. 4

(c) Deduce the correct expressions for Gibbs free energy and Helmholtz free energy. 3

8. (a) In a two dimensional gas, the molecules can move freely on a plane, but are constrained within an area $A$. Show that, 3+2+2

(i) The density of states is given by, $g(E)\,dE = \dfrac{2\pi mA}{h^2} dE$.

(ii) The single particle partition function is given by, $Z_1 = \left(\dfrac{2\pi mA}{h^2}\right) kT$.

(iii) The equation of state is given by, $p = \dfrac{NkT}{A}$.

(b) $N$ distinguishable particles are distributed in three states having energy 0, kT and 3 kT. If the total equilibrium energy of the system is 2000 kT, then find $N$. 3

9. (a) Draw the FD distribution function for temperatures $T = 0\,\mathrm{K}$ and $T \neq 0\,\mathrm{K}$. 2

(b) Show that for a completely degenerate Fermi system, the Fermi energy is given by, $E_\mathrm{F} = \dfrac{h^2}{8m}\left(\dfrac{3n}{\pi}\right)^{2/3}$, where $n$ is the number density of the fermions. 3

(c) Explain Bose-Einstein condensation. Derive an expression for the critical temperature at which this phenomenon sets in. 5

10. (a) What is thermodynamic probability? 1

(b) Find the expression of thermodynamic probability for a macro-state. 3

(c) Calculate the percentage error introduced in using Stirling's approximation when $n$ = 5, 10 and 20. Hence comment on the result. What would be the case for $n \to \infty$? 3+1+1

(d) What is Boson? 1

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