UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2020

CC2-MATHEMATICS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** the following questions: 2×6 = 12

(a) Prove that $|z| \geq \text{Re}(z)$, the equality occurs when z is a non-negative real number.

(b) Find the real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in $[0, 1]$.

(c) Use Euclidean algorithm to find the integers u and v such that $\gcd(72, 120) = 72u + 120v$.

(d) Show that the mapping $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by, $f(x) = x^3 - x$, $x \in \mathbb{R}$ is surjective.

(e) Separate $(a + ib)^i$, where $i = \sqrt{-1}$, into real and imaginary parts.

(f) If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$, for $x \in \mathbb{R}$, then find $f^{-1}(-8)$.

GROUP-B

2. Answer ***all*** the following questions: 5×4 = 20

(a) Show that the point a^z lies on the equiangular spiral $r = \sigma^{\frac{1}{u}(u^2+v^2)} \cdot e^{-(\frac{v}{u})\theta}$, where $z = u + iv$, $a = \sigma(\cos\psi + i\sin\psi)$, $-\pi \leq \psi \leq \pi$.

(b) Show that the number of reflexive relations on a set of n-elements is 2^{n^2-n}.

(c) Find for what values of a and b the following system of equations has (i) a unique solution, (ii) no solution, (iii) infinite number of solutions over the field of rational numbers:

$$x_1 + 4x_2 + 2x_3 = 1$$
$$2x_1 + 7x_2 + 5x_3 = 2b$$
$$4x_1 + ax_2 + 10x_3 = 2b + 1$$

(d) Solve the equation $3x^4 + x^3 + 4x^2 + x + 3 = 0$, which has four distinct roots of equal moduli.

GROUP-C

3. Answer ***all*** the following questions: 7×4 = 28

(a) (i) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum \alpha^2\beta^2$. 3+4

(ii) If an equation contains only odd powers of the variable and if the co-efficients are positive, show that it has no real root except zero.

(b) (i) Let $f: A \rightarrow B$ be a mapping. A relation ρ is defined on A by "$x \rho y$ if and only if $f(x)=f(y)$, $x, y \in A$". Show that ρ is an equivalence relation. 3+4

(ii) If X and Y are two non-empty sets and $f: X \rightarrow Y$ is a mapping, then for any subsets A and B of X, prove that

$$f(A \cup B) = f(A) \cup f(B)$$

(c) (i) Find the row space and column space of the following matrix: 4+3

$$\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$$

(ii) Find the remainder when $1! + 2! + 3! + ... + 100!$ is divided by 15.

(d) (i) If α, β, γ are the roots of the equation $x^3+qx+r=0$ $(r \neq 0)$, find the equation whose roots are $\frac{\beta}{\gamma}+\frac{\gamma}{\beta}$, $\frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}$, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$. 4+3

(ii) z is a variable complex number such that $\left|z-\frac{10}{z}\right|=3$. Find the greatest and the least value of $|z|$.

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