UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2020

CC6-MATHEMATICS

GROUP THEORY-I

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** the following questions: $2\times6 = 12$

(a) Find the image of the elements 3 and 4 if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & & & 3 \end{pmatrix}$ be an odd permutation.

(b) Give an example of a group of order 4 which is non-cyclic.

(c) In a group $(G, \circ)$, a is an element of order 30. Find the order of a^{18}.

(d) Find the number of elements of order 6 in S_4.

(e) If $G = \langle a \rangle$ is a cyclic group of order 40, find all the distinct elements of the cyclic subgroup $\langle a^{10} \rangle$.

(f) Find all distinct left cosets of the subgroup $H = \{1, -1\}$ in the group $G = (R\{0\}, \cdot)$.

GROUP-B

2. Answer ***all*** the questions from the following: $5\times4 = 20$

(a) If an abelian group of order six contains an element of order 3, show that it must be a cyclic group.

(b) If p is a prime number and G is a non-abelian group of order p^3, show that the centre of G has exactly p elements.

(c) Show that the four permutations $I,\ (ab),\ (cd),\ (ab)(cd)$ on four symbols a, b, c, d form a finite abelian group with respect to the permutation multiplication.

(d) Use Lagrange's Theorem to prove that a finite group cannot be expressed as the union of two of its proper subgroups.

GROUP-C

3. Answer ***all*** the following questions: 7×4 = 28

(a) Prove that for a square with centre *O*, four symmetrics arising for rotation of the square in the plane about *O* and four symmetrics arising for rotation out of the plane together form a non-commutative group. 7

(b) Prove that the normaliser of a subgroup *H* of a group *G* is a subgroup of *G* and also prove that the subgroup *H* is a normal subgroup of the normaliser of *H*. 4+3

(c) (i) Prove that if *G* is an abelian group, then for all $a, b \in G$ and integers *n*, $(ab)^n = a^n b^n$. 4+3

(ii) If *H* be a subgroup of a group *G* and $T = \{x \in G : xH = Hx\}$, prove that *T* is a subgroup of *G*.

(d) If $G = \{a\}$ be a finite cyclic group of order *n*, then prove that for any divisor *d* of *n*, there exists a subgroup of *G* of order *d* and also prove that subgroup will be unique. 4+3

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