SUKANTA MAHAVIDYALAYA

B.Sc. 2nd Semester Internal Examination-2021
DSC & GE - MATHEMATICS
ALGEBRA

Full Marks: 30

ASSIGNMENT

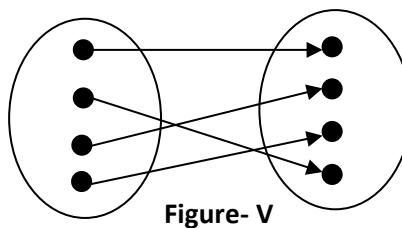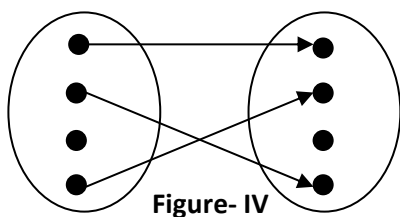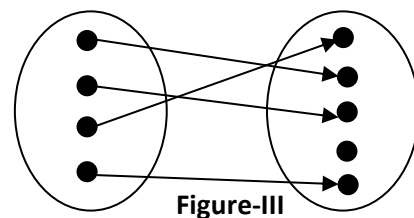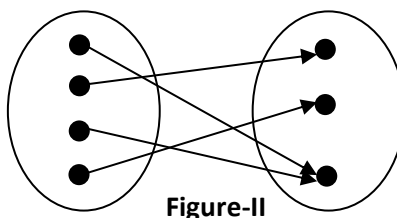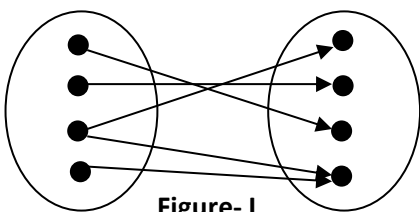Answer ***all*** questions: 3+4+3+5+5+5+5=30

1. If $\alpha = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ and p is prime to n, prove that
$$1 + \alpha^p + \alpha^{2p} + \cdots + \alpha^{(n-1)p} = 0.$$

2. If α, β, γ be the roots of the equation $x^3 - 2x^2 + 3x - 1 = 0$, find the equation whose roots are $\frac{\beta\gamma - \alpha^2}{\beta + \gamma - 2\alpha}, \frac{\gamma\alpha - \beta^2}{\gamma + \alpha - 2\beta}, \frac{\alpha\beta - \gamma^2}{\alpha + \beta - 2\gamma}$.

3. Solve the equation $x^4 + 32x - 60 = 0$ by Ferrari's method.

4. What kind of mappings are represented by following figures and why?

Figure- I   Figure-II   Figure-III

Figure- IV   Figure- V

5. Show that $3^{4n+2} + 5^{2n+1}$ is divisible by 14. Where n is any natural number.

6. Determine the conditions for which the following system of equations has
   i) Only one solution;
   ii) No solution;
   iii) Many solutions;
$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$

7. Find the eigen values and the corresponding eigen vectors of
$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

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