UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2021

CC4-COMPUTER SCIENCE (23)

DISCRETE STRUCTURES

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.

Answer any *three* questions from the following 20×3 = 60

1. (a) Explain injective, surjective and bijective mappings with examples. 10

(b) Prove by induction that $1+3+5+\cdots+(2n-1)=n^2$ for all positive integer *n*. 5

(c) Let *n* be a positive integer and consider the following algorithm segment: 5

for *i* **:=** 1 to *n*

for *j* := 1 to *i*

[*Statements in body of inner loop*.

None contain branching statements that lead outside the loop.]

next *j*

next *i*

Discuss complexity of this algorithm segment.

2. (a) Explain Equivalence Relation with proper examples. 5

(b) Solve the recurrence relation $a_k = 3a_{k-1}$ for $k = 1, 2, 3, \cdots$ and initial condition $a_0 = 2$, using Generating function. 5

(c) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages? 5

(d) Answer the following: 5

(i) How many distinguishable ordering are there of the letters of ALGEBRA?

(ii) How many distinguishable orderings of the letters of ALGEBRA contain the expression “AAB”?

3. (a) Let $A=\{0, 1, 2\}, B=\{3, 4, 5, 6\}, C=\{7, 8, 9\}$ and $U=\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Evaluate the following: 10

i. $A \cup B$ ii. $A \cap B$ iii. $A \setminus B$ iv. $B \setminus A$ v. $A \Delta B$

vi. $C \cup B$ vii. $C \cap B$ viii. $C \setminus B$ ix. $B \setminus C$ x. C^c

(b) How many different five-digit numbers can be formed from the digits 1, 2, 3, 4, 5 where 5

(i) No restrictions on digits and repetitions allowed.

(ii) The number is odd and no repetitions are allowed.

(iii) The number is even and no repetitions are allowed.

(c) Solve linear recurrence relation: $a_n = 3a_{n-1} + 2^n$ with condition $a_0 = 1$. 5

4. (a) If $f : Z \; X \; Z \to Z$, where Z is the set of integers and $f(x, y) = x * y = x + y - xy$. 5

Prove that the binary operation * is commutative and associative. Find the identity element.

(b) Explain big O, big theta (Θ) and big omega (Ω) notations used in complexity analysis of algorithms. 9

(c) Seven women and nine men are on the faculty in the mathematics department at a college. 6

(i) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

(ii) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

5. (a) Discuss Master Theorem. 10

(b) Prove by induction that $1^2 + 2^2 + 3^2 + \cdots n^2 = n(n+1)(2n+1)/6$ for all n. 5

(c) Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men? 5

6. Write short notes on the following: 5×4=20

(a) Partial Order

(b) Complexity of Insertion Sort

(c) Bounding Summations

(d) Transitive Closure.

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