UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2021

MATHEMATICS

INTEGRAL CALCULUS-II AND DYNAMICS OF A PARTICLE

PAPER-VI

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer *all* questions

1. (a) Let $f:[0,1]\longrightarrow \mathbb{R}$ be such that $f(x)=x$ for x rational number and $f(x)=0$ for x irrational number. Evaluate the upper and lower integrals of f and show that f is not integrable. 5

   (b) Using Riemann criterion, examine whether the function $f(x)=\frac{1}{x}$ is integrable on [0, 2] or not. 5

GROUP-B

Answer *all* questions

2. (a) If time t be regarded as a function of velocity v, then prove that the rate of decrease of acceleration is given by $f^3\frac{d^2t}{dv^2}$. 5

   (b) A particle moving in a plane besides the central acceleration P, an acceleration T perpendicular to P is acting on it. Show that in usual notation the differential equation of the path is $\frac{d^2u}{d\theta^2}+u=\frac{P-\frac{T\,du}{u\,d\theta}}{h^2-u^2}$. 5

3. (a) A particle rests in equilibrium under the attraction of two centers of force which attract directly as the distance, their intensities being μ and μ'. The particle is slightly displaced toward one of them, show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu+\mu'}}$. 5

(b) If the radial and transverse velocity of a particle be always proportional to each other, then show that the path is an equiangular spiral. 5

4. (a) A particle is projected vertically upwards with a velocity v_0, in a resisting medium which produces a retardation kv^2 when the velocity is v. Show that the particle comes to rest at a height $\frac{V^2}{2g}\log_e\left(1+\frac{v_0^2}{V^2}\right)$ above the point of projection where V is the terminal velocity. Show further that the velocity v_1 of the particle when it reaches the point of projection is given by $\frac{1}{v_1^2}=\frac{1}{v_0^2}+\frac{1}{V^2}$. 6

(b) A particle describes the curve $r^2=a^2\cos^2\theta+b^2\sin^2\theta$ under an attraction to the origin. Prove that the attraction at a distance r is $h^2\{2(a^2+b^2)r^2-3a^2b^2\}r^{-7}$, where symbol h has its usual meaning. 4

5. (a) A boat which is rowed with constant velocity u, starts from a point A on the bank of a river which flows with a constant velocity v and it points always towards a point B on the other bank exactly opposite to A. Find the equation of the path of the boat. If $u=v$, then show that the path is a parabola whose focus is B. 6

(b) If T be the time taken by a heavenly body to describe an arc of a parabolic orbit bounded by the focal chord, then show that $T \propto (\text{focal chord})^{3/2}$. 4

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