UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2021

MATHEMATICS

PAPER-VII

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer *all* questions 5×3 = 15

1. If G be an infinite cyclic group generated by a, then prove that a and a^{-1} are the only generators of G.

2. Let H be a subgroup of the symmetric group S_n. Show that either every member of H is an even permutation or exactly half of them are even.

3. A ring R is called a Boolean ring if $a^2 = a$, $\forall a \in R$.
   Prove that (i) $\mathbb{Z}_2 \times \mathbb{Z}_2$ is a Boolean ring.
   (ii) The characteristic of a Boolean ring is 2.

GROUP-B

Answer *all* questions 5×3 = 15

4. The matrix of a linear mapping $T:\ \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ relative to the ordered basis $\{(-1, 1, 1), (1, -1, 1), (1, 1, -1\}$ of $\mathbb{R}^3$ is $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$. Find the matrix of T relative to the ordered basis $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of $\mathbb{R}^3$.

5. Apply Gram-Schmidt process to obtain an orthonormal basis of the subspace of Euclidean space of $\mathbb{R}^4$ with standard inner product, spanned by the vectors $(1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, 2)$.

6. Find the dimension of the subspace $S \cap T$ of $\mathbb{R}^4$ where

$$S = \{(x,\ y,\ z,\ w) \in \mathbb{R}^4 :\ x + y + z + w = 0\},$$
$$T = \{(x,\ y,\ z,\ w) \in \mathbb{R}^4 :\ 2x + y - z + w = 0\}$$

GROUP-C

Answer *all* questions 10×2 =20

7. (i) Show that the vector $\vec{a}\times(\vec{b}\times\vec{a})$ is coplanar with $\vec{a}$ and $\vec{b}$. 1+2+3+4

(ii) Determine the constant a so that the vector

$$\vec{v}=(x+3y)\hat{i}+(y+2z)\hat{j}+(x+az)\hat{k}$$

is solenoidal.

(iii) Find the maximum value of the directional derivative of $\varphi=x^2-y^2+z^2$ at the point (1, 3, 2). Find also the direction in which it occurs.

(iv) Find the curvature and torsion for the curve $x=a\cos t$, $y=a\sin t$, $z=bt$ at any point t.

8. (i) Verify Stoke's theorem for the vector function $\mathbf{F}=(x^2-y^2)\hat{i}+2x\hat{j}$ round the rectangle bounded by the straight lines $x=0$, $x=a$, $y=0$, $y=b$. 5+5

(ii) Find the equations of the osculating plane, normal plane and rectifying plane to the twisted cubic $x=2t$, $y=t^2$, $z=\frac{1}{3}t^3$ at $t=1$.

——×——