UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2021

CC3-MATHEMATICS

REAL ANALYSIS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer *all* questions 2×5 = 10

1. (a) Find the derived set of the set $A=(0,2)\cup(1,3)\cap Q$, where Q is the set of rational numbers. 2

(b) Find all limit points of the sequence $(\sin n)_{n\in\mathbb{N}}$ 2

(c) Find a bijection from Z^+ to $Z^+\times Z^+$ where Z^+ is the set of all positive integers. 2

(d) Construct a sequence $(r_n)_{n\in\mathbb{N}}$ of rational numbers that converges to a given real number r. 2

(e) Examine if for any $A\subset\mathbb{R}$, $\overline{A}=\{x\in A\,;\ \exists$ a sequence (x_n) in A so that $x_n\to x\}$. 2

GROUP-B

Answer *all* questions 10×3 = 30

2. (a) Prove that the series $\dfrac{1}{x+1}+\dfrac{x}{x+2}+\dfrac{x^2}{x+3}+........(x>0)$ converges if $x<1$ and diverges if $x\geq 1$. 5

(b) If $\sum\limits_{n=1}^{\infty}a_n^2$ is convergent, prove that $\sum\limits_{n=1}^{\infty}\dfrac{a_n}{n}$ is also convergent ($a_n>0\ \ \forall n\in\mathbb{N}$). 5

3. (a) Show that $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}\cdots\cdots=\ln 2$. 5

(b) Find $\left(1-\dfrac{1}{2}\right)+\left(\dfrac{1}{3}-\dfrac{1}{4}\right)\cdots\cdots=?$ 5

4. (a) Show that finite union of compact subsets of $\mathbb{R}$ is compact. What about infinite union in this regard? 5+2

(b) Show that arbitrary intersection of compact subsets of $\mathbb{R}$ is compact. 3

GROUP-C

Answer *all* questions 5×2 = 10

5. Check if the family of all finite subsets of the set of natural numbers is countable. 5

6. Check if the family $\zeta = \left\{\left(r_n - \frac{1}{2^{n+1}},\ r_n + \frac{1}{2^{n+1}}\right);\ n \in \mathbb{N}\right\}$ is an open cover of $\mathbb{R}$ where $(r_n)_{n\in\mathbb{N}}$ is a linear array of all rational numbers. 5

GROUP-D

Answer *all* questions 5×2 = 10

7. Let the sequence (x_n) of real numbers converges to the real number x and $p:\mathbb{N} \longrightarrow \mathbb{N}$ is a bijection. Check if $x_{p(n)} \to x$. 5

8. Let $f : D \longrightarrow \mathbb{R},\ D \subset \mathbb{R}$ be a continuous function and (x_n) be a sequence in D.

(a) Examine if $(f(x_n)) \to f(x)$ if $x_n \to x \in D$. 3

(b) Examine if $(f(x_n))$ is a Cauchy sequence if (x_n) is Cauchy. 2

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