UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2021

MATHEMATICS

PAPER-IX

LINEAR PROGRAMMING AND OPTIMIZATION, TENSOR ALGEBRA AND ANALYSIS

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer *all* the questions 10×3 = 30

1. (a) Prove that $x_1 = 2$, $x_2 = 1$ and $x_3 = 3$ is a feasible solution of the set of equations $4x_1 + 2x_2 - 3x_3 = 1$, $-6x_1 - 4x_2 + 5x_3 = -1$. Reduce the feasible solution to a basic feasible solution by reduction theory. 5+3+2

   (b) Solve graphically the L.P.P.

$$\begin{array}{rl} \text{Maximize} & z = 2x_1 + x_2 \\ \text{Subject to} & x_1 + x_2 \leq 2 \\ & -x_1 + x_2 \leq 1 \\ & x_1 \leq 2 \\ \text{and} & x_1,\ x_2 \geq 0 \end{array}$$

   (c) Find the dual of the following L.P.P.

$$\begin{array}{rl} \text{Maximize} & z = 4x_1 - 7x_2 \\ \text{Subject to} & 3x_1 + x_2 \leq 16 \\ & x_1 - 2x_2 \leq 12 \\ & x_1 \geq 2 \\ & x_2 \geq 4 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{array}$$

2. (a) Prove that the set $X = \{(x,\ y) \in \mathbb{R}^2 :\ xy \leq 1,\ \ x \geq 0,\ y \geq 0\}$ is not convex. 2+2+6

   (b) Find out the extreme points (if any) of the convex set

$$X = \{(x,\ y) \in \mathbb{R}^2 :\ x^2 + y^2 \leq 16\}$$

(c) Obtain the optimal solution of the following transportation problem:

	A	B	C	D	E	a_i
I	4	1	3	4	4	60
II	2	3	2	2	3	35
III	3	5	2	4	4	40
b_j	22	45	20	18	30	

3. (a) Solve the following L.P.P. by Big-M method: 5+5

Maximize $z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$,

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$,

$x_1,\ x_2,\ x_3 \geq 0$

(b) Solve the following assignment problem:

	P	Q	R	S	T
I	9	8	7	6	4
II	5	7	5	6	8
III	8	7	6	3	5
IV	8	5	4	9	3
V	6	7	6	8	5

GROUP-B

Answer *all* the questions 5×4 = 20

4. (a) If the intrinsic derivative of non-null vector $\vec{A}$ along a curve is zero, show that the magnitude of $\vec{A}$ is constant along the curve. 2+3

(b) Show that a second order covariant tensor can be expressed as a sum of a symmetric and a skew symmetric tensor.

5. If the metric is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$, then find the conjugate metric tensor. 5

6. (a) Prove that $A^{ij}_{j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j}\left(A^{ij}\sqrt{g}\right) + A^{jp}\begin{Bmatrix} i \\ j \quad p \end{Bmatrix}$, where A^{ij} is a tensor of type (2, 0). 3+2

(b) If $R_{ij,\,k} + R_{jk,\,i} + R_{ki,\,j} = 0$, prove that the scalar curvature R is constant.

8. Surface of a sphere is a two dimensional Riemannian space. Compute the Christoffel symbols. 5

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