



**UNIVERSITY OF NORTH BENGAL**  
B.Sc. Honours Part-III Examination, 2021

**MATHEMATICS**

**PAPER-X**

Full Marks: 50

**ASSIGNMENT**

*The figures in the margin indicate full marks.  
All symbols are of usual significance.*

**GROUP-A**

**Answer all the questions**

5×5 = 25

1. (a) Examine if an uncountable subset of irrational numbers may be compact. 3
- (b) Let  $f : (a, b) \rightarrow [c, d]$  and  $g : [a, b] \rightarrow (c, d)$  be surjections. Examine continuity of the functions  $f$  and  $g$ . 2
  
2. Let  $f : D \rightarrow \mathbb{R}$ ,  $D \subset \mathbb{R}$  be a mapping such that for any  $\varepsilon > 0$  and  $a \in D$ ,  $\exists \delta_a > 0$  such that  $|f(x) - f(a)| < \varepsilon$ ,  $\forall x \in D$  satisfying  $|x - a| < \delta_a$ . Let  $\delta = \inf \{\delta_a; a \in D\}$ . Examine uniform continuity of  $f$  in its domain  $D$  if  $\delta > 0$ . If  $f$  is uniformly continuous in  $D$ , examine if  $\delta > 0$  necessarily. 2+3
  
3. Let  $f$  and  $\{f_n : n \in \mathbb{N}\}$  be continuous real valued functions on  $[0, 1]$ .  
(a) If  $\{f_n\}$  converges uniformly to  $f$ , show that for each  $k \in \{0, 1, 2, \dots\}$  3  
$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) x^k dx = \int_0^1 f(x) x^k dx.$$
  
(b) Prove that the converse of the statement in 3(a) does not hold in general. 2
  
4. Prove that the series  $\sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{x^2}{1 + \lambda^2} \right)^{\frac{1}{n}}$  is uniformly convergent in  $\mathbb{R}$ . 5
  
5. Find the point on the  $x$ -axis which minimizes the sum of the squares of the distances from  $p$  to  $(0, 0)$  and from  $p$  to  $(3, 2)$ . 5

**GROUP-B**

Answer *all* the questions

5×5 = 25

6. Determine if the integral  $\int_{-\infty}^0 \frac{e^{1/x}}{x^2} dx$  converges or diverges. If the integral converges determine its value. 3+2
7. Compute:  $\int_0^{\pi/2} \cos^m \theta \sin^n \theta d\theta$  5
8. (a) Give an example of a real valued function of real variable which cannot be expanded as a Fourier Series. (Provide reason) 2  
 (b) Find the value to which the sum of Fourier series of  $f(x)$  converges at the point of discontinuity  $x = a$ . 3
9. (a) Find the volume of the tetrahedron bounded by the coordinate planes and the plane  $z = 4 - 4x - 2y$ . 3  
 (b) Find the volume of the solid bounded above by the plane  $z = 4 - y$  and below by the region  $D$  enclosed within the circle  $x^2 + y^2 = 4$ . 2
10. Evaluate :
- (a)  $\int_0^1 \frac{\tan^{-1} \lambda x}{x\sqrt{1-x^2}} dx$  2
- (b)  $\int_0^1 \frac{\tan^{-1} \lambda x}{\sqrt{1-x^2}} dx$  3

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