



**UNIVERSITY OF NORTH BENGAL**

B.Sc. Honours Part-III Examination, 2021

**MATHEMATICS**

**PAPER-XII**

**THEORY OF PROBABILITY AND RIGID DYNAMICS**

Full Marks: 50

**ASSIGNMENT**

*The figures in the margin indicate full marks.  
All symbols are of usual significance.*

**Answer all questions**

**GROUP-A**

1. (a) A unbiased die is thrown 1200 times. Find the minimum value of the probability of getting 160 to 240 sixes. 3

- (b) If the joint pdf of the random variable  $X, Y$  is 2+3+2

$$f_{x,y} = \begin{cases} k(3x+y) & , \quad 0 \leq x < 3, \quad 0 \leq y \leq 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find (i)  $P(X+Y < 2)$  (ii) The marginal distribution of  $X$  and  $Y$ . Investigate whether  $X$  and  $Y$  are independent.

2. (a) The pdf of a random variable  $X$  is symmetric about the origin. Prove that  $X$  and  $-X$  have the same distribution. 4

- (b) If  $(X, Y)$  is a standard normal variable in two dimensions and  $\rho(X, Y) = \rho$ , then prove that  $\rho(X^2, Y^2) = \rho^2$ . 6

3. For two arbitrary events  $A$  and  $B$  defined on the event space  $W$ , show that 3+2  
 $P(B|A) \geq 1 - \frac{P(A)}{P(B)}$ ,  $P(A) \neq 0$ . Draw the probability distribution curve of uniform distribution defined by the pdf

$$f_x(x) = \begin{cases} \frac{1}{b-a} & , \quad a \leq x < b \\ 0 & , \quad \text{otherwise} \end{cases}$$

4. If  $n$  coins are distributed among  $m$  beggars at random, what is the probability that one of the beggars will get exactly  $k$  coins? 5

5. Find the constant  $k$  such that the function  $f$  given by 1+2+2

$$f_x(x) = \begin{cases} k|x| & , \quad -2 \leq x < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

is a possible pdf and find its distribution function and compute  $P(X > 1)$ .

**GROUP-B**

6. Find the moment of inertia of a solid ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  about any principal axis. 5

7. A rod, of length  $2a$ , is suspended by a string, of length  $l$ , attached to one end; if the string and rod revolved about the vertical with uniform angular velocity, and their inclination to the vertical be  $\theta$  and  $\phi$  respectively, show that 5

$$\frac{3l}{a} = \frac{(4 \tan \theta - 3 \tan \phi) \sin \phi}{(\tan \phi - \tan \theta) \sin \theta}$$

8. An elastic ball of mass  $m$  falls from a height  $h$  on a fixed horizontal plane and rebounds. Show that the loss of kinetic energy by the impact is  $mgh(1 - e^2)$ ,  $e$  being the coefficient of restitution. 5

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