UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2021

MATHEMATICS

PAPER-XIII

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** the questions: $5\times5 =25$

   (a) The distribution of a population random variable X is given by $P(X=0)=P(X=1)=\dfrac{1}{2}$. A random sample (x_1, x_2, x_3, x_4) of size 4 is taken from the hypothetical population of X. Show that the sampling distribution of the statistic $t = x_1 + x_2 + x_3 + x_4$ is a binomial $(4, \frac{1}{2})$. 5

   (b) The mean of 5 observations is 4.4 and the variance is 8.24. If three of the five observations be 1, 2 and 6, find the other two. 5

   (c) In a random sample of 1000 persons taken from a state A, 510 are found to be smokers. In another sample of 800 persons taken from a state B, 480 are found to be smokers. Do these facts reveal a significant difference between the states A and B as far as smoking habit is concerned? 5

   (d) Find a curve of the form $y = x^2 + ax + b$ to the following data by the method of least squares: 5

x	2	3	4	5	6
y	7.2	3.9	3	4.4	6.3

   (e) (i) A random sample of size 25 is taken from a $N(\mu, \sigma^2)$ with $\mu = 30$ and $\sigma^2 = 16$. Would the probability that the sample mean would be between 25 and 35 be greater than 0.99? 2

   (ii) If T_1, T_2, T_3 are independent unbiased estimators of θ and all have the same variance, which of the following estimator of θ will you prefer? 3

   $$(T_1 + 2T_2 + T_3)/4, \quad (2T_1 + T_2 + 2T_3)/5, \quad (T_1 + T_2 + T_3)/3$$

GROUP-B

Answer *all* the questions
$5\times5 =25$

2. (a) Define Poinsot's central axis.   1

  (b) If the curved surface of a hemisphere rests in equilibrium on a sphere of equal radius, then show that the equilibrium is unstable.   2

  (c) Prove that the reaction R of any smooth surface with which a body is in contact does not work for a small displacement.   2

3.   Find the c.g. of area enclosed by the curves $y^2 = ax$ and $x^2 + y^2 = 2ax$, lying in the first quadrant.   5

4.   If two equal uniform rods connected by a smooth hinge are placed over the circumference of a smooth vertical circle, apply the principle of virtual work to show that in the position of equilibrium the inclination of either rod to the vertical is given by $\operatorname{cosec}^2\theta \cot\theta = \dfrac{l}{a}$, where $2l$ and $2a$ are the length of a rod and the diameter of the circle respectively.   5

5. (a) A wire 140 yds. long hangs between two points, 138 yds. apart horizontally and 50 ft vertically. Show that the tension at its lowest point is about 495 lbs.wt., the wire weighing half a pound per foot.   2

  (b) Forces X, Y, Z act along the three straight lines $y = b, z = -c$; $z = c, x = -a$; $x = a$, $y = -b$, respectively; show that they will have single resultant if   3

$$\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$$

6.   Prove that if equal forces act along the edges BC, CA, AB, DA, DB, DC of a regular tetrahedron, the central axis is the perpendicular from D to the plane ABC and the pitch of the equivalent wrench is $\dfrac{a}{2\sqrt{2}}$, where 'a' is an edge of the tetrahedron.   5

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