UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2021

MATHEMATICS

PAPER-XIV

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** questions: 15

   (a) What do you understand by generalised coordinate? Express velocity and acceleration in terms of the generalized coordinate system. 5

   (b) State the *principle of virtual work*. Two heavy particles of weights W and w are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius R, the axis of which is horizontal. Find the condition of equilibrium of the system by applying the *principle of virtual work*. 5

   (c) Show that the following force is conservative and find the corresponding potential $F_r = ar^2 \cos\theta$, $F_\theta = ar^2 \sin\theta$, $F_z = 2az^2$, where a is a constant. 5

GROUP-B

2. Answer ***all*** questions: 20

   (a) Let A be some fixed 10-element subset of {1, 2, 3, …. , 50}. Using Pigeonhole principle show that A possesses two different 5-element subsets, the sums of whose elements are equal. 5

   (b) Find the explicit formula for the following linear homogeneous recurrence relation: 3

   $$3a_n = 7a_{n-1} - 2a_{n-2} , \quad \text{if} \quad n > 1$$

   with the initial conditions $a_0 = -2$ and $a_1 = 1$.

   (c) Prove that $(D_{20}, \leq)$ is a lattice, where D_{20} = set of all positive divisors of 20 and $\leq$ is the divisibility relation. Hence, find the complement of 10 in this lattice. 4

   (d) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same colour? 4

(e) Given the Boolean expression   4

$$X = AB + ABC + A\overline{B}\,\overline{C} + A\overline{C}$$

    (i)   Draw the logic diagram for the expression.

    (ii) Minimize the expression and draw the logic diagram.

GROUP-C

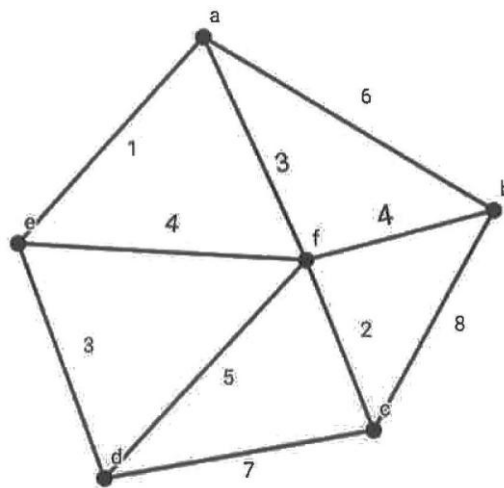5.   Answer *all* questions: 15

  (a) If G is connected simple planar graph with $n\,(\geq 3)$ vertices and e edges then show   5
that $e \leq 3n-6$ and additionally if it is with no circuits of length 3, then show that
$e \leq 2n-4$.

  (b) Use *Prim's algorithm* to find a *minimal spanning tree* of the graph shown.   4

  (c) Show that the *complete graph* $K_n\,(n > 2)$ always contains a *Hamiltonian cycle*   3
but contains an *Euler circuit* if n is odd.

  (d) Let G be a graph of order n that has exactly two connected components, both of   3
them being complete graphs. Prove that the size of G is at least $\dfrac{n^2 - 2n}{4}$.

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