UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2021

CC14-MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

Answer *all* questions from the following

GROUP-A

1. Answer ***all*** questions: 2×5 = 10

(a) Obtain the partial differential equation for $Z = f(\sin x + \cos y)$.

(b) Solve $\sqrt{p} + \sqrt{q} = 1$, where symbols have their usual meaning.

(c) State whether the following partial differential equations are linear, quasi-linear or nonlinear:

(i) $U_{xx} + U_{yy} + \log U = 0$

(ii) $U_{xx} + 2U_{xy} + U_{yy} = \sin x$

(d) Find the general integral of the linear partial differential equation

$$x\,(x^2 + 3y^2)\,p - y\,(3x^2 + y^2)\,q = 2z(y^2 - x^2)$$

(e) Obtain the solution of the linear partial differential equation $U_x - U_y = 1$ with the Cauchy data $U(x, 0) = x^2$.

GROUP-B

2. (a) Use separation of variables $U(x,\ y) = f(x)g(y)$ to solve the equation 4+6

$$y^2U_x^2 + x^2U_y^2 = (xyu)^2$$

(b) Find the solution of the initial value problem,

$$u_{tt} = c^2u_{xx}\ ,\quad x \in R,\quad t > 0 \quad \text{and} \quad u(x, 0) = \log(1 + x^2),\quad u_t(x,\ 0) = 2$$

3. Classify the following equations and reduce it to its canonical form 5+5

(a) $U_{xx} - (\sec h^4 x)\, U_{yy} = 0$

(b) $\sin^2 x \dfrac{\partial^2 z}{\partial x^2} + \sin 2x \dfrac{\partial^2 z}{\partial x \partial y} + \cos^2 x \dfrac{\partial^2 z}{\partial y^2} = x$

4. (a) Solve by method of characteristic for $x > 0$, $xU_x + yU_y = xe^{-u}$, $u(x, x^2) = x$. 5+5

(b) Solve: $(y^3 x - 2x^4)\, p + (2y^4 - x^3 y)\, q = 9z(x^3 - y^3)$

GROUP-C

5. (a) Determine the solution of the initial boundary-value problem 5+5

$$U_{tt} = 16\, U_{xx} \quad , \quad 0 < x < \infty \; , \quad t > 0$$
$$U(x, 0) = \sin x \quad , \quad 0 \le x \le \infty$$
$$U_t\,(x, 0) = x^2 \quad , \quad 0 \le x \le \infty$$
$$U(0, t) = 0 \quad , \quad t \ge 0$$

(b) Solve by method of separation of variables for $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

GROUP-D

6. (a) Apply $\sqrt{U} = V$ and $V(x, y) = f(x) + g(y)$ to solve the equation 5+5

$$x^4 U_x^2 + y^2 U_y^2 = 4U$$

(b) Find the solution of the initial value system

$$U_t + 3UU_x = V - x \;,\; V_t - cV_x = 0 \quad \text{with} \quad U(x, 0) = x \quad \text{and} \quad V(x, 0) = x$$

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