'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2021

CC1-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer any ***five*** questions from the following: $1\times5 = 5$

(a) Calculate, $\lim\limits_{x\to1}\dfrac{x^2-1}{x-1}$.

(b) Find out whether $\sin(\omega t)$ and $\cos(\omega t)$ can be two solutions of a second order homogeneous ordinary differential equations.

(c) Evaluate $\oiint_S \vec{r}\cdot d\vec{s}$ where *S* is the surface enclosing a volume *V*.

(d) Find '*a*' such that the vector $\vec{F}$
$\vec{F}=(4x+3y)\hat{i}+(y+2z)\hat{j}+(x+az)\hat{k}$ is solenoidal.

(e) Show that the function
$$f(x,y) = \begin{cases} x^2+2y & (x,y)\neq(1,2) \\ 0 & (x,y)=(1,2) \end{cases}$$
is discontinuous at (1, 2)

(f) What is the greatest rate of increase of $u=xyz^2$ at $(1,0,3)$?

(g) If $\vec{\nabla}\times\vec{A}=\dfrac{\partial\vec{B}}{\partial t}$, then show that $\vec{\nabla}\cdot\vec{B}$ is independent of *t*.

(h) Determine the order of the differential equation $\dfrac{d^3y}{dx^3}-15\dfrac{dy}{dx}=e^x+2$.

GROUP-B

Answer any *three* questions from the following $5\times3 = 15$

2. Solve the differential equation $\dfrac{d^2y}{dx^2}-8\dfrac{dy}{dx}+15y=0$ 5

3. Verify Gauss divergence theorem for a vector $\vec{V}=\hat{r}/r^2$, the region of integration being a sphere of radius *R* with centre at the origin. 5

4. Prove that $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=0$ in any orthogonal curvilinear coordinate system. 5

5. (a) Find the directional derivative of the divergence of $\vec{F}=xy\hat{i}+xy^2\hat{j}+z^2\hat{k}$ at the point (3, 1, 4) in the direction of outwardly directed normal to the sphere $x^2+y^2+z^2=4$. 3

(b) Evaluate, $\vec{\nabla}\times(\phi\,\vec{\nabla}\phi)$. 2

6. (a) If A and B are two events with $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{3}$, $P(A\cup B)=\frac{1}{2}$. Find $P(A\,|\,B)$, $P(B\,|\,A)$. 3

(b) Two players A and B play a game such that the player A has probability $\frac{2}{3}$ of winning whenever it plays. If A plays 4 games, find the probability that A wins exactly 2 games. 2

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. (a) Evaluate $[\vec{\nabla}\cdot(r^n\vec{r})]$. Show that $r^n\vec{r}$ is solenoidal for $n=-3$. 6

(b) If $\phi(x,y,z)=3x^2y-y^3z^2$, find $\vec{\nabla}\phi$ at the point $(1,-3,-1)$. 4

8. (a) Show that $\oint \vec{r}\cdot d\vec{s}=3V$, where V is the volume enclosed by the closed surface S. 3

(b) Obtain the expression for $\nabla^2\psi$ in spherical polar coordinates. 7

9. (a) Express $z\hat{i}-2x\hat{j}+y\hat{k}$ in cylindrical coordinates. 5

(b) Prove that $2\frac{1}{2}+2\frac{1}{2}$

(i) $\delta(x)=\delta(-x)$

(ii) $f(x)\,\delta(x-a)=f(a)\,\delta(x-a)$

10. (a) Obtain the expression for Variance of Poisson Distribution. 2

(b) Using Lagrange's multiplier method, show that the rectangle of maximum area that can be inscribed in a circle is a square. 4

(c) Show that the functions $e^{ax}\sin bx$ and $e^{ax}\cos bx$ are linearly independent with the help of Wronskian. 4

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