'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2021

CC11-MATHEMATICS

GROUP THEORY-II

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) Find the number of elements of order 7 in $\mathbb{Z}_4 \times \mathbb{Z}_7$. 3

(b) Examine if direct product of two cyclic group is cyclic. 3

(c) Examine if every group of order p^n $(n \geq 1)$ is normal where p is any prime. 3

(d) Write class equation of S_3. 3

(e) Find the number of Sylow 2-subgroups of S_4 and A_4. 3

(f) If $f : G \to G$ defined by $f(x) = x^2$ for all $x \in G$ is an automorphism does it imply that G is commutative? 3

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

2. Find all groups of order 22. 6

3. Let G be a finite group and $a \in G$. Prove that $[G ; C(a)] = |Cl(a)|$, where $C(a)$ is the centralizer of a and $Cl(a)$ is the conjugacy class of a. 6

4. Prove or disprove: A_5 is a simple group. 6

5. Show that any group of order pq where p, q are primes, $p > q$ and q does not divide $p-1$ is cyclic. 6

6. Show that every group of order 14 contains 6 elements of order 7. 6

7. Prove that direct product of two finite cyclic groups is cyclic if and only if orders of the cyclic groups are relatively prime. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) State and prove Cauchy's theorem. 2+6

(b) Find smallest n such that the Klein's four group is isomorphic to a subgroup of S_n. 4

9. (a) Let G be an infinite cyclic group. Prove that Aut(G) is isomorphic to $\mathbb{Z}_2$. 6

(b) Find Aut($\mathbb{Q}$), where $\mathbb{Q}$ is the group of rational numbers. 6

10.(a) Let G be a group and H, K be two subgroups of G. If G is an internal direct product of H and K, then prove that 6

(i) $G \cong H \times K$ (ii) $G/H \cong K$ and (iii) $G/K \cong H$

(b) Let H and K be any two Sylow p-subgroups of a finite group G. Prove that $H = xKx^{-1}$ for some $x \in G$. 6

11.(a) Let $f : G \to G$ be a homomorphism. Suppose f commutes with every inner automorphism of G. Show that 8

(i) $K = \{x \in G \; ; \; f^2(x) = f(x)\}$ is a normal subgroup of G

(ii) G/K is abelian.

(b) Show that characteristic subgroup of a group is normal. 4

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