'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2022

CC3-MATHEMATICS

REAL ANALYSIS

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following — 3×4 = 12

1. Prove that $\mathbb{N} \times \mathbb{N}$ is an enumerable set. — 3

2. Examine whether the sequence $\left\{1+\frac{1}{1!}+\frac{1}{2!}+\ldots\ldots+\frac{1}{n!}\right\}$ is a Cauchy sequence. — 3

3. When is a series of constant terms called conditionally convergent? Give an example. — 3

4. Is arbitrary union of compact sets a compact set? Justify your answer. — 3

5. Check whether the set $\{-1+\frac{1}{n},\ n\in\mathbb{N}\}$ is closed or not. — 3

6. Find $\overline{\lim}\, U_n$ and $\underline{\lim}\, U_n$, where $U_n = n^{(-1)^n}$. — 3

GROUP-B

Answer any *four* questions from the following — 6×4 = 24

7. Show that the sequence $\sqrt{a},\ \sqrt{a\sqrt{a}},\ \sqrt{a\sqrt{a\sqrt{a}}},\ \ldots\ldots\ (a>0)$ is convergent. — 6

8. (a) Give example of two distinct sets A and B such that $\operatorname{int} A = \operatorname{int} B$. — 3

   (b) Prove that the set $S=\{x\in\mathbb{R}:\ \sin x \neq 0\}$ is an open set. — 3

9. Define compact set. Prove that closed and bounded subset of real numbers is compact. — 6

10. (a) Define derived set. Obtain derived set of $\left\{\frac{(-1)^m}{m}+\frac{1}{n}:\ m,\ n\in\mathbb{N}\right\}$. — 4

    (b) Prove that $\sqrt{2}$ is not a rational number. — 2

11. Use comparison test to prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$ and diverges if $p \leq 1$. 6

12.(a) Use Cauchy's criterion to prove that the sequence $\left\{1+\frac{1}{2}+\frac{1}{3}+ \ldots\ldots + \frac{1}{n}\right\}$ does not converge. 4

(b) Find all the sub-sequential limits of the sequence $\{\sin \frac{n\pi}{3}\}$. 2

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Let $\sum_{n=1}^{\infty} U_n$ be a positive term series such that $\lim_{n\to\infty} \sqrt[n]{U_n} = l$. Prove that the series converges if $l<1$ and diverges if $l>1$. 5

(b) If $a_n > 0 \;\; \forall n$. Show that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ converge or diverge together. 3

(c) Test the convergence of the series $\frac{2^2}{3^2} + \frac{2^2.4^2}{3^2.5^2} + \frac{2^2.4^2.6^2}{3^2.5^2.7^2} + \ldots\ldots$. 4

14.(a) Show that the set of real numbers $\mathbb{R}$ is uncountable. 6

(b) Show that the derived set of any bounded set is also bounded set. 6

15.(a) Prove that every bounded sequence of real numbers has a convergent sub-sequence. 5

(b) Prove that the sequence $\{x_n\}$, where $x_1 = \sqrt{7}$ and $x_n = \sqrt{7+x_{n-1}}$ for $n = 2, 3, 4, \ldots$, is convergent. 4

(c) If $\{x_n\}$ is a sequence of positive real numbers converging to l, then show that $\lim_{n\to\infty} \sqrt[n]{x_1 x_2 \ldots\ldots x_n} = l$. 3

16.(a) Let $S = \{x : x \in \mathbb{Q}$ and $x^2 < 2\}$, where $\mathbb{Q}$ is the set of all rational numbers. Show that $\sup S \notin \mathbb{Q}$. 6

(b) Let $S = (0, 1]$ and $T = \{\frac{1}{n} : n \in \mathbb{N}\}$. Show that $S - T$ is an open set. 6

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