'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2022

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following $3\times4 = 12$

1. Verify if $\exp(x)$ and $\exp(2x)$ are independent functions. 3

2. Solve: $\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2+1=0$ 3

3. If $\vec{r}=\cos(nt)\hat{i}+\sin(nt)\hat{j}$, where n is a constant and t varies. Show that $\vec{r}\times\dfrac{d\vec{r}}{dt}=nk$. 3

4. Find the directional derivative of $\phi=xy^2z+4x^2z$ at (–1, 1, 2) along the direction $(2\hat{i}+\hat{j}-2\hat{k})$. 3

5. Find the unit tangent vector at the point where $t=2$ on the curve $x=t^2+1$, $y=4t-3$, $z=2t^2-6t$. 3

6. If $y=\exp(-x^2)$ is a solution of $xy''+\alpha y'+\beta x^3y=0$ for any two real numbers α, β then find $\alpha\beta$. 3

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

7. Solve: $(D^2-4D+4)y=x^2+e^x+\sin(2x)$ 6

8. Find the workdone in moving a particle around a circle in xy plane if the circle has centre at origin and radius 3 and the force field is given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. 6

9. Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $r=|\vec{r}|$ and $f(r)$ is a scalar function possessing first and 2nd order derivatives prove that $\nabla^2 f(r)=\dfrac{d^2f}{dr^2}+\dfrac{2}{r}\dfrac{df}{dr}$. 6

10. Show that $\dfrac{\partial^2 \vec{f}}{\partial x \partial y} = \dfrac{\partial^2 \vec{f}}{\partial y \partial x}$, where $\vec{f} = (2x^2 y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + x^2 \cos y\hat{k}$. 6

11. Find the characteristic roots of the following system and hence solve it. 6

$$\dot{x} = 3x + 2y$$
$$\dot{y} = -5x + y$$

12. Solve Euler's equation: $(x+1)^2 y'' + (x+1)y' - y = 0$ 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Find three independent solutions of $x^3 \dfrac{d^3 y}{dx^3} - 6x\dfrac{dy}{dx} + 12y = 0$. 6

(b) Show that for a differentiable function $f(r)$ one must have $\operatorname{curl}\{f(r)\vec{r}\} = \vec{0}$, where $r = |\vec{r}|$. 6

14.(a) Prove that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is a conservative force field. Find the scalar potential V such that $\vec{F} = \nabla V$. 6

(b) A particle p is moving on a circle of radius r with constant angular velocity $\omega = d\theta/dt$. Show that the acceleration is $-\omega^2 \vec{r}$. 6

15.(a) Solve the differential equation by the method of undetermined co-efficients 6

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$

(b) Prove that a necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant direction is $\vec{f} \times \dfrac{d\vec{f}}{dt} = \vec{0}$. 6

16.(a) Obtain expressions for radial and transverse velocities of a moving particle in a plane and hence show that the radial and transverse accelerations are $\dfrac{d^2 r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2$ and $\dfrac{1}{r}\dfrac{d}{dt}\left(r^2 \dfrac{d\theta}{dt}\right)$. 6

(b) Solve: $(x+2)^2 \dfrac{d^2 y}{dx^2} + (2+x)\dfrac{dy}{dx} + 4y = 2\sin\{2\log(x+2)\}$ 6

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