UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2022

CC8-Physics

Mathematical Methods-III

Time Allotted: 2 Hours   Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1.   Answer any *five* questions from the following: 1×5 = 5

(a) Which of the following is analytic function of the complex variable $z = x + iy$:

   (i) $|z|$ (ii) $\text{Re}(z)$ (iii) $\sin z$ (iv) $\log z$

(b) What kind of singularity exists for the function $f(z) = \dfrac{\sin z}{z}$ at $z = 0$?

(c) Solve: $x^5 = 1$

(d) If A is an orthogonal matrix then show that , $|A| = \pm 1$.

(e) Is it possible to have Fourier transformation of a function $f(x)$ having infinite periodicity? Justify your answer.

(f) Write down the polar form of Cauchy-Riemann conditions.

(g) Find the transpose of the following matrix:

$$\begin{bmatrix} 1 & i & 3 \\ -i & 2i-4 & 0 \\ 0 & -i & 1 \end{bmatrix}$$

(h) Find the residue of $e^{-\frac{1}{z}}$ at the singularity.

GROUP-B

Answer any *three* questions from the following   5×3 = 15

2. (a) Find the Fourier transform of the function   3

$$f(x) = \begin{cases} 1 & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$$

(b) Hence show that, $\int\limits_{0}^{\infty} \left(\dfrac{\sin u}{u} \right)^{2} du = \dfrac{\pi}{2}$.   2

3.   Given $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ then find the matrix,   5

$$A^{100} + A^{99} + A^{98} + \cdots\cdots + A^{2} + A$$

4. (a) Evaluate the following integral using Cauchy's integral formula: 3

$$\oint\limits_{C} \dfrac{e^{2z}}{(z+1)^{4}}\, dz$$

where C is the circle $|z| = 3$.

(b) Find the Laurent series of $f(z) = \dfrac{z - \sin z}{z^{3}}$ about the singularity $z = 0$.   2

5.   For the given matrix   5

$$M = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$$

Find the eigen vector associated with each eigen values.

6. (a) Find the principal value of i^{i}.   2

(b) Show that, the function $x^{2} + iy^{3}$ is not analytic anywhere.   3

GROUP-C

Answer any *two* questions from the following $10 \times 2 = 20$

7.   Given the matrix $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,

(a) Diagonalize the matrix B.   4

(b) Using Cayley-Hamilton's theorem, find the characteristic equation satisfied by the matrix B.   4

(c) Are the eigenvectors of the matrix B linearly independent?   1

(d) Are the eigenvectors orthogonal to each other?   1

8. (a) Define essential singularity, removable singularity and simple pole of order n with proper examples.   3

    (b) Evaluate the following integral   6

$$I = \int\limits_{0}^{\infty} \frac{dx}{x^6 + 1} \text{ using the residue theorem.}$$

    (c) What kind of singularity exist for the function $f(z) = \dfrac{1}{\sqrt{z}}$?   1

9. (a) Find the Fourier transformation of the function $f(x) = e^{-|x|}$ and its inverse transformation.   6

    (b) From the previous function, prove that $\int\limits_{0}^{\infty} \dfrac{\cos \alpha x}{\alpha^2 + 1} d\alpha = \dfrac{\pi}{2} e^{-|x|}$.   4

10.(a) Let $f(z) = u + iv$ be an analytic complex function, then show that u and v are harmonic function.   2

    (b) Find the poles and the residues at the poles of the function $f(z) = \left(\dfrac{z+1}{z-1} \right)^2$.   4

    (c) Evaluate the integral, $\int\limits_{0}^{2\pi} \dfrac{d\theta}{5 + 4\sin\theta}$.   4

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